Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-10 (2016) 252–257

Design of a switched robust control scheme for drug delivery in blood pressure regulation ⋆

Saeed Ahmed * **Hitay Özbay** **

* *Department of Electrical and Electronics Engineering, Bilkent University, Ankara 06800, Turkey. (e-mail: ahmed@ee.bilkent.edu.tr)*
** *Department of Electrical and Electronics Engineering, Bilkent University, Ankara 06800, Turkey. (e-mail: hitay@bilkent.edu.tr)*

Abstract: A control algorithm based on switching robust controllers is presented for a Linear Parameter Varying (LPV) time-delay system modeling automatic infusion of vasodilator drug to regulate postsurgical hypertension. The system is scheduled along a measurable signal trajectory. The prospective controllers are robustly designed at various operating points forming a finite set of robust controllers and then a hysteresis switching is performed between neighboring robust controllers for a larger operating range of the LPV system. The stability of the switching LPV system for the entire operating range is ensured by providing a sufficient condition in terms of bound on the scheduling signal variation using the concept of dwell time. Simulation results are provided to verify the performance of the designed control scheme.

Keywords: Time-delay system, LPV system, Uncertain system, Robust control, Hysteresis switching, Biomedical control system, Life sciences

1. INTRODUCTION

The regulation of postoperative hypertension is essential during general clinical and operative scenarios to decrease bleeding. It becomes particularly vital for postoperative cardiac patients suffering from Myocardinal Revascularization for a quick recovery because they do not possess an autonomic capability of regulating their increased blood pressure and an external infusion of a vasodilator drug (a drug facilitating blood flow due to decrease in vascular resistance) is needed to reduce their high blood pressure, see Mitchell (1982) and Koch-Weser (1974) for more details.

A formal research for the development of automatic control schemes for regulation of postoperative hypertension dates back to the late 1970's when Slate et al. (1979) presented an experimentally validated mathematical model relating the patient's blood pressure response to the injection of Sodium Nitroprusside (SNP) drug. The blood pressure response of the model was in agreement with the response observed in actual postsurgical patients. From 1970's through 1980's, many contributions were made towards fixed gain Proportional-Integral-Derivative (PID) controllers for postoperative hypertension regulation as in Sheppard et al. (1979), Smolen et al. (1979), De Asla et al. (1985) and Pardini et al. (1988). From late 1980's, there has been a remarkable shift in trend towards the use of adaptive controllers for blood pressure regulation. The adaptive controllers found in literature for blood pressure regulation can be classified as: Self Tuning Regulators (STR), Model Reference Adaptive Controllers (MRAC) and Multiple Model Adaptive Controllers (MMAC). See Isaka and Sebald (1993) for a complete summary of these adaptive controllers and their application for the regulation of blood pressure.

Recently, Luspay and Grigoriadis (2015) introduced the concept of LPV control for regulation of postsurgical hypertension. They used a Multiple Model Extended Kalman Filter (MMEKF) algorithm for online estimation of blood pressure response model parameters and a LPV control algorithm for the regulation of blood pressure. Ahmed and Özbay (2015) proposed switching PI Smith-predictor based robust controllers for a LPV time-delay system modeling automatic administration of SNP drug in postsurgical scenario. This paper is an extension of our previous work. In this work, we provide a finite dimensional approximation of the original LPV time-delay model representing the blood pressure response to drug infusion. We also provide a sufficient condition for stability of the switching LPV finite-dimensional approximated system based on the idea of Yan and Özbay (2007).

The rest of the paper is organized as follows. In Section 2, mathematical description of the process, design constraints and formulation of the LPV framework are presented. In Section 3, the switching robust control scheme is given for the LPV system under consideration. Section 4 discusses the stability of the switching LPV system. Finally, Section 5 presents simulation results to verify the performance of our proposed control algorithm.

⋆ This work is supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) under project EEEAG-115E820.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.538

2. PROBLEM DEFINITION

2.1 Model description

The model relating patient's blood pressure response to the infusion of a vasodilator drug is given by the continuous-time, third order, stable, time-delayed transfer function $\Sigma(s)$ as

$$\Sigma(s) := \frac{\Delta P_d(s)}{I(s)} = \frac{K(\tau_3 s+1)e^{-Ts}}{(\tau_1 s+1)[(\tau_2 s+1)(\tau_3 s+1)-\alpha]} \quad (1)$$

where $I(s)$ is the Laplace transform of the drug delivery rate in $\frac{ml}{h}$ and $\Delta P_d(s)$ is the Laplace transform of the change in blood pressure in $mmHg$. In (1), K is the patient's sensitivity to the drug in $mmHg\left(m\ h^{-1}\right)^{-1}$, T is the initial injection delay in seconds, α is the drug fraction recirculating and, finally τ_1, τ_2 and τ_3 are the time constants in seconds for vasodilator drug action, flow through pulmonary circulation and flow through systemic circulation, respectively. The Mean Arterial Pressure (MAP) is given as

$$MAP(t) = \Delta P_d(t) + P_0 \quad (2)$$

where $P_0 = 150\ mmHg$ is the initial blood pressure, which is known and fixed.

This model, adopted from Martin et al. (1987), is a variant of empirically validated model of Slate et al. (1979). The model consists of three first-order sections depicting drug action, systemic circulation and pulmonary circulation as shown in Fig. 1. Later, this model was also adopted by Malagutti et al. (2013) and Malagutti (2014) for their research work.

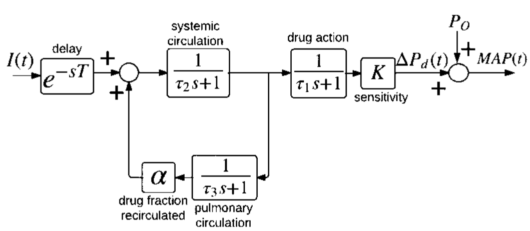


Fig. 1. Compartmental model proposed by Martin et al. (1987)

It has been shown by Wood et al. (1987) that the patient's sensitivity to the drug varies not only from patient to patient but also within the individual patient. Therefore, treating the variability in intra-patient response to the drug infusion, we consider the patient's sensitivity to the drug as a time-varying measurable signal, $K(t) \in [-9.5, -0.25]\ mmHg\left(m\ h^{-1}\right)^{-1}$. A MMEKF algorithm can be employed for an online estimate of $K(t)$ as shown in Luspay and Grigoriadis (2014). Treating the variability in inter-patient response to the drug injection, drug fraction recirculating $\alpha \in [0.25, 0.65]$ and initial injection delay $T \in [20, 60]\ sec$ are considered to be uncertainties of considerably large and known ranges. The ranges of these uncertainties are also in accordance with the clinical validated data, Meijers et al. (1997). The nominal values of α and T are taken to be $\alpha_0 = 0.5$ and $T_0 = 50\ sec$, respectively, see Slate et al. (1979) and Malagutti et al. (2013) for more discussion. We assume the time constants $\tau_1 = 50\ sec$, $\tau_2 = 30\ sec$ and $\tau_3 = 10\ sec$ to be known and fixed, Martin et al. (1987).

2.2 Design Constraints and Performance Specifications

Our main aim is to reduce the blood pressure from an initial value of 150 $mmHg$ to a final value of 100 $mmHg$ and maintain this level within $\pm 5\ mmHg$ of final value considering uncertainties in T and α, and time-variation in measurable scheduling signal $K(t)$. Based on our earlier work Ahmed and Özbay (2015) and Malagutti (2014), the design constraints and performance specifications for postsurgical hypertension regulation problem are listed below:

- The maximum settling time should preferably be $\leq 10\ min$ but it must not exceed 15 min,
- MAP should be within $[70, 120]$ $(mmHg)$ once it settles to this interval in order to be in agreement with the normal physiological blood pressure limits,
- MAP should be within $\pm 5\ mmHg$ of 100 $mmHg$ during steady state,
- MAP should not drop below the danger threshold of 70 $mmHg$,
- The acceptable range of vasodilator SNP drug injection is $0 \leq I(t) \leq 180\ ml\ h^{-1}$ due to toxic side effects of SNP,
- The response of system must not be oscillatory or unstable at anytime.

2.3 Formulation of the LPV framework

Considering the range of uncertainties in model parameters and to fulfill the performance specifications, we have chosen a third-order Padé approximation (3) to insert the time delay into the model dynamics.

$$e^{-sT} \approx P_{T,3}(s) = \frac{1 - Ts/2 + (Ts)^2/10 - (Ts)^3/120}{1 + Ts/2 + (Ts)^2/10 + (Ts)^3/120} \quad (3)$$

Therefore, a finite-dimensional approximation of (1) can be modeled as

$$\Sigma_a(s) := \frac{K(\tau_3 s+1)P_{T,3}(s)}{(\tau_1 s+1)[(\tau_2 s+1)(\tau_3 s+1)-\alpha]} \quad (4)$$

Considering $K(t)$ to be a measurable scheduling signal, and defining input variable as $u(t) = I(t)$ and output variable as $y(t) = \Delta P_d(t)$, we can formulate a LPV framework for the model given in (4) as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\ \mathbf{x}(t) + \mathbf{B}(K(t))\ u(t) \\ y(t) &= \mathbf{C}\ \mathbf{x}(t) \end{aligned} \quad (5)$$

where $\mathbf{x}(t)$ is the state vector defined as

$$\mathbf{x}(t) = [x_1(t)\ x_2(t)\ x_3(t)\ x_4(t)\ x_5(t)\ x_6(t)]^T .$$

The system matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -a_6 & -a_5 & -a_4 & -a_3 & -a_2 & -a_1 \end{bmatrix}$$

where $a_1 = \frac{12}{T} + \Upsilon$, $a_2 = \frac{6}{T^2} + \frac{12}{T}\Upsilon + \phi$,
$a_3 = \frac{120}{T^3} + \frac{6}{T^2}\Upsilon + \frac{12}{T}\phi + \psi$, $a_4 = \frac{120}{T^3}\Upsilon + \frac{6}{T^2}\phi + \frac{12}{T}\psi$,
$a_5 = \frac{120}{T^3}\phi + \frac{6}{T^2}\psi$, $a_6 = \frac{12}{T^3}\psi$,
with
$\Upsilon = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3}$, $\phi = \frac{1}{\tau_1\tau_3} + \frac{1}{\tau_1\tau_2} + \frac{1-\alpha}{\tau_2\tau_3}$, $\psi = \frac{1-\alpha}{\tau_1\tau_2\tau_3}$.

The input matrix $\mathbf{B}(K(t))$ is given by

$$\mathbf{B}(K(t)) = [\beta_1 \ \beta_2 \ \beta_3 \ \beta_4 \ \beta_5 \ \beta_6]$$

where $\beta_1(K(t)) = 0$, $\beta_2(K(t)) = -\frac{K(t)}{\tau_1\tau_2}$,
$\beta_3(K(t)) = b_3(K(t)) - a_1\beta_2(K(t))$,
$\beta_4(K(t)) = b_4(K(t)) - a_1\beta_3(K(t)) - a_2\beta_2(K(t))$,
$\beta_5(K(t)) = b_5(K(t)) - a_1\beta_4(K(t)) - a_2\beta_3(K(t)) - a_3\beta_2(K(t))$,
$\beta_6(K(t)) = b_6(K(t)) - a_1\beta_5(K(t)) - a_2\beta_4(K(t)) - a_3\beta_3(K(t)) - a_4\beta_2(K(t))$
with
$b_3(K(t)) = \frac{12K(t)}{\tau_1\tau_2 T} - \frac{K(t)}{\tau_1\tau_2\tau_3}$,
$b_4(K(t)) = \frac{12K(t)}{\tau_1\tau_2\tau_3 T} - \frac{60K(t)}{\tau_1\tau_2 T^2}$,
$b_5(K(t)) = \frac{120K(t)}{\tau_1\tau_2 T^3} - \frac{60K(t)}{\tau_1\tau_2\tau_3 T^2}$,
$b_6(K(t)) = \frac{120K(t)}{\tau_1\tau_2\tau_3 T^3}$.

The output matrix is $\mathbf{C} = [1 \ 0 \ 0 \ 0 \ 0 \ 0]$.

Therefore, the state-space model formulated in (5) is a LPV system, where $K(t)$ is the measurable scheduling signal. Note that for a fixed $K(t) = K$, we have LTI model of (1) and for a particular trajectory of $K(t)$, we have an LTV system.

In this paper, assuming the knowledge of uncertainty range of T and α, and assuming $K(t)$ to be a measurable time-varying signal with a known range, we perform hysteresis switching between neighboring controllers within a finite set of robust controllers for a larger operating range of the LPV system using the idea of Yan and Özbay (2007) as discussed in Section 3. Note that a single robust controller cannot fulfill the performance specifications and design constraints discussed in Section 2.2 due to considerably large uncertainty range of the model parameters, Ahmed and Özbay (2015).

3. CONTROL ALGORITHM

Our finite-dimensional LPV system formulated in (5) depends on measurable scheduling signal $K(t)$. We assume that $K(t) \in \mathbb{R}$ is continuously differentiable and $K(t) \in \mathcal{K}$ where $\mathcal{K}$ is a compact set, Yan and Özbay (2007).

Similar to our earlier work, Ahmed and Özbay (2015), we select six different subsets (we will call these subsets as operating ranges) of the notably large range of the measurable scheduling signal $K(t) \in [-9.5, -0.25]$ as given below:

$\mathcal{K}_1 = [K_1^-, K_1^+] = [-9.50, -4.50]$ for controller C_1;
$\mathcal{K}_2 = [K_2^-, K_2^+] = [-4.50, -2.30]$ for controller C_2;
$\mathcal{K}_3 = [K_3^-, K_3^+] = [-2.30, -1.30]$ for controller C_3;
$\mathcal{K}_4 = [K_4^-, K_4^+] = [-1.30, -0.80]$ for controller C_4;
$\mathcal{K}_5 = [K_5^-, K_5^+] = [-0.80, -0.45]$ for controller C_5;
$\mathcal{K}_6 = [K_6^-, K_6^+] = [-0.45, -0.25]$ for controller C_6.

We formulate a set of six robust controllers C_1 through C_6 designed for the operating ranges mentioned above at operating points $K = K_1 = -7$, $K = K_2 = -3.4$, $K = K_3 = -1.8$, $K = K_4 = -1.05$, $K = K_5 = -0.625$ and $K = K_6 = -0.39$ respectively, and perform hysteresis switching between neighboring robust controllers, which provides a larger operating range of the LPV system as shown in the Fig. 2.

Let us denote the operating ranges as $\mathcal{K}_\eta$ for $\eta = 1, 2, ..., 6$ and operating points as K_η for $\eta = 1, 2, ..., 6$, the prospective controllers are chosen from a controller set $\mathcal{C} := \{C_\eta(s) : \eta = 1, 2, ..., 6\}$, where $C_\eta(s)$ is an LTI robust controller designed for $K = K_\eta$ for $\eta = 1, 2, ..., 6$, as in Yan and Özbay (2007).

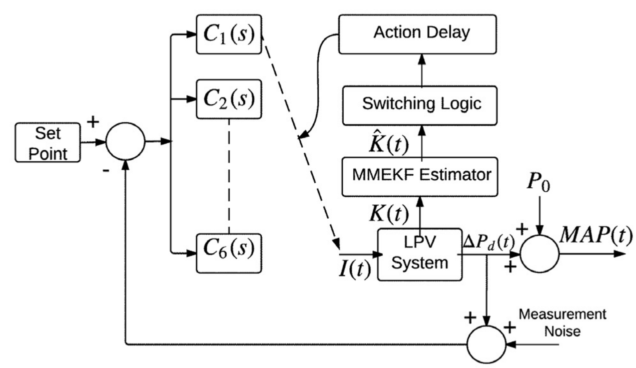


Fig. 2. Switching LPV system

We use the notation $\mathcal{L}\{f(t,K)|_{K=K_0}\} = f_{K_0}(s)$ to represent (5) in Laplace domain at fixed operating points K_η for $\eta = 1, 2, ..., 6$, by which nominal transfer function $P_{K_\eta}(s)$ for $\eta = 1, 2, ..., 6$ can be represented as below:

$$P_{K_\eta}(s) = \frac{K_\eta(\tau_3 s + 1)P_{T_0,3}(s)}{(\tau_1 s + 1)[(\tau_2 s + 1)(\tau_3 s + 1) - \alpha_0]} \qquad (6)$$

where $P_{T_0,3}(s) = P_{T,3}(s)\ |_{T=T_0}$.

Defining uncertainty as:

$$\Delta_{m\eta}(s) = \frac{P_{K_\eta}(s) - P(s)}{P_{K_\eta}(s)}, \quad \forall P(s) \in \mathcal{P}_\eta(s) \qquad (7)$$

for $\eta = 1, 2, ..., 6$, where $\mathcal{P}_\eta(s)$ represent the set of all uncertain plants corresponding to an operating range $\mathcal{K}_\eta$ as given below:

$$\mathcal{P}_\eta(s) = \left\{ \frac{\tilde{K}(\tau_3 s + 1)e^{-Ts}}{(\tau_1 s + 1)[(\tau_2 s + 1)(\tau_3 s + 1) - \alpha]} : \begin{array}{r} \tilde{K} \in \mathcal{K}_\eta, T \in [20, 60], \\ \alpha \in [0.25, 0.65], \tau_1 = 50, \\ \tau_2 = 30, \tau_3 = 10. \end{array} \right. \qquad (8)$$

for $\eta = 1, 2, ..., 6$.

Choosing an upper bound weight $W_{2\eta}(s)$ for $\eta = 1, 2, ..., 6$ on the uncertainty defined in (7) such that

$$|\ \Delta_{m\eta}(j\omega)\ | < |\ W_{2\eta}(j\omega)\ | \quad \forall\ \omega \qquad (9)$$

for $\eta = 1, 2, ..., 6$.

Let us assume that the desired nominal complimentary sensitivity function is in the form

$$T_\eta(s) = Q_\eta(s) P_{K\eta}(s) \text{ for } \eta = 1, 2, ..., 6. \quad (10)$$

The five robust candidate controllers can be constructed using controller parameterization, Ozbay (1999):

$$C_\eta(s) = \frac{Q_\eta(s)}{1 - Q_\eta(s) P_{K\eta}(s)} \text{ for } \eta = 1, 2, ..., 6. \quad (11)$$

Then, in order to achieve the performance specifications listed in Section 2.2, $T_\eta(s) = Q_\eta(s) P_{K\eta}(s)$, for $\eta = 1, 2, ..., 6$, should be in the following desired form:

$$T_\eta(s) = Q_\eta(s) P_{K\eta}(s) = \frac{P_{T_0,3}(s)}{1 + \tau_{d\eta}{}^{(1)} s + \tau_{d\eta}{}^{(2)} s^2} \quad (12)$$

for $\eta = 1, 2, ..., 6$.

where $\tau_{d\eta}{}^{(1)} > 0$ and $\tau_{d\eta}{}^{(2)} > 0$ for $\eta = 1, 2, ..., 6$, are to be designed.

From (12), we have

$$Q_\eta(s) = \frac{K_\eta{}^{-1}(\tau_1 s + 1)[(\tau_2 s + 1)(\tau_3 s + 1) - \alpha_0]}{(\tau_3 s + 1)(1 + \tau_{d_\eta}{}^{(1)} s + \tau_{d_\eta}{}^{(2)} s^2)} \quad (13)$$

for $\eta = 1, 2, ..., 6$.

where $\tau_{d\eta}{}^{(1)} > 0$ and $\tau_{d\eta}{}^{(2)} > 0$ for $\eta = 1, 2, ..., 6$, are to be designed so that the robust stability condition (14) is satisfied.

$$\left\| \frac{W_{2\eta}(s)}{1 + \tau_{d\eta}{}^{(1)} s + \tau_{d\eta}{}^{(2)} s^2} \right\|_\infty < 1 \text{ for } \eta = 1, 2, ..., 6. \quad (14)$$

For time-domain performance constraints listed in section 2.2, we choose the smallest possible $\tau_{d\eta}{}^{(1)} > 0$ and $\tau_{d\eta}{}^{(2)} > 0$ satisfying (14). For instance, in order to design the controller C_1 in the operating range $\mathcal{K}_1 = [-9.5, -4.5]$ at the operating point $K_1 = -7$, we can select a weight $W_{21}(s) = 0.95 + \frac{1.45(40s+0.001)(40s+2.5)}{1600s^2+40(2+\sqrt{3})s+7.3}$ with $\tau_{d_1}{}^{(1)} = 33\ sec$ and $\tau_{d_1}{}^{(2)} = 1\ sec$ in order to satisfy (9) and (14) as shown in Fig. 3. A similar analysis holds for rest of the controllers in $\mathcal{C}$.

Finally, using (13) in (11), we obtain

$$C_\eta(s) = \frac{K_\eta{}^{-1}(\tau_1 s + 1)[(\tau_2 s + 1)(\tau_3 s + 1) - \alpha_0]}{(\tau_3 s + 1)[1 + \tau_{d_\eta}{}^{(1)} s + \tau_{d_\eta}{}^{(2)} s^2 - P_{T_0,3}(s)]} \quad (15)$$

for $\eta = 1, 2, ..., 6$.

4. STABILITY OF SWITCHING LPV SYSTEM

Using the main idea of Yan and Özbay (2007), we will prove the stability of our switching LPV system shown in Fig. 2. For this purpose, first we will show that $C_\eta(s)$ robustly stabilizes $P_{K_\eta}(s)$ for $\eta = 1, 2, ..., 6$. A sufficient condition to ensure that $C_\eta(s)$ robustly stabilizes $P_{K_\eta}(s)$ for $\eta = 1, 2, ..., 6$ is given by Zhou et al. (1996) and Ozbay (1999), which is presented below:

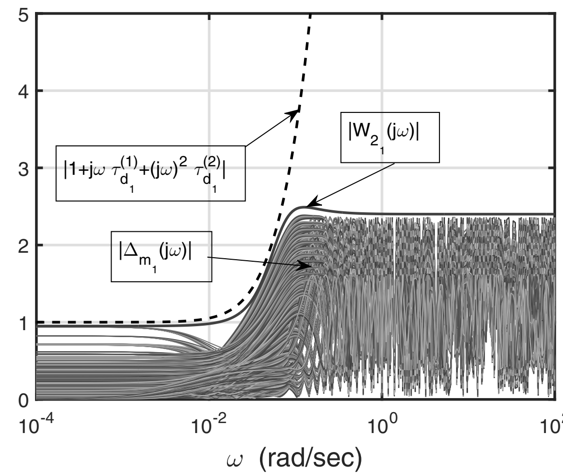


Fig. 3. Selection of uncertainty bounds, $W_{2_1}(s)$, $\tau_{d_1}{}^{(1)}$ and $\tau_{d_1}{}^{(2)}$

$$\xi := \left\| W_{2\eta}(s) P_{K_\eta}(s) C_\eta(s)[1 + P_{K_\eta}(s) C_\eta(s)]^{-1} \right\|_\infty \leq 1 \quad (16)$$

for $\eta = 1, 2, ..., 6$.

For instance, we can choose a weight $W_{21}(s) = 0.95 + \frac{1.45(40s+0.001)(40s+2.5)}{1600s^2+40(2+\sqrt{3})s+7.3}$ along with $\tau_{d_1}{}^{(1)} = 33\ sec$ and $\tau_{d_1}{}^{(2)} = 1\ sec$ so that (16) is satisfied for controller C_1. Thus, C_1 robustly stabilizes $P_{K_1}(s)$ as shown in Fig. 4. A similar analysis holds for rest of the controllers in $\mathcal{C}$. Thus, $C_\eta(s)$ stabilizes $P_{K_\eta}(s)$ for $\eta = 1, 2, ..., 6$ which can be ensured with a proper choice of $K_\eta{}^-$ and $K_\eta{}^+$ such that $K \in \mathcal{K}_\eta = [K_\eta{}^-, K_\eta{}^+]$.

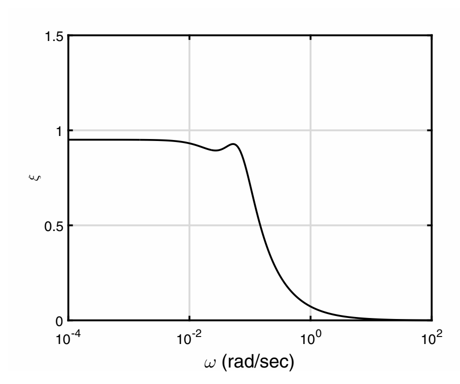


Fig. 4. $\mathcal{H}^\infty$ bound test

For a notably large operating range $\mathcal{K}$, we devise hysteresis switching scheme over $\mathcal{C}$ using Yan and Özbay (2007) and try to find a bound on $|\dot{K}(t)|$ to ensure the stability of the switching LPV system shown in Fig. 2 over the controller set $\mathcal{C}$. A necessary condition for stable switching, Yan and Özbay (2007), is

$$\mathcal{K} \subseteq \bigcup_{\eta=1}^{6} \mathcal{K}_\eta \quad (17)$$

A sufficient condition for the hysteresis switching to ensure stability of switching LPV system of Fig. 2 in terms of bound on $|\dot{K}(t)|$ over the robust controller set $\mathcal{C}$ with operating range $\mathcal{K}_\eta$ obeying (17) is given by *Corollary 3.1* of Yan and Özbay (2007), which is presented below:

$$|\dot{K}(t)| < \min_{\eta \in \mathcal{F}} \left\{ \frac{|d_{\eta,\eta+1}|}{\tau_D} \right\} \tag{18}$$

where $d_{\eta,\eta+1} = \mathcal{K}_\eta \cap \mathcal{K}_{\eta+1}$ is the ηth hysteresis interval as shown in Fig. 5, τ_D is the dwell time. The set $\mathcal{F} = \{1, 2..., 6\}$ is such that $\eta \in \mathcal{F}$, see Yan and Özbay (2007) for more details.

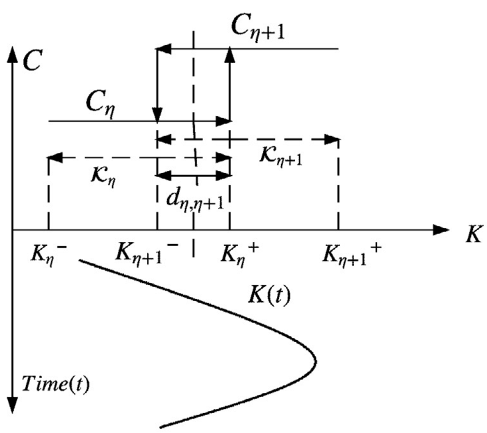


Fig. 5. Hysteresis Switching

Corollary 3.1 of Yan and Özbay (2007) is based on dwell time based switching concept i.e. the distance between any consecutive switchings should be larger than τ_D, which is a constant. Note that (18) follows from the fact that in worst case of switching i.e. when $K(t)$ fluctuates around the midpoint of the interval $d_{\eta,\eta+1}$ with an amplitude $|d_{\eta,\eta+1}|/2$, the condition $|\dot{K}(t)| < |d_{\eta,\eta+1}|/\tau_D$ is sufficient to guarantee the stability of the system, Yan and Özbay (2007).

In the next section, we will provide simulation results to verify the performance of our switching robust control scheme under various operating conditions.

5. SIMULATIONS

In this section, we implement our designed switching robust control scheme in MATLAB for the LPV system (5) in order to satisfy the performance specifications subject to the design constraints mentioned in Section 2.2. For this purpose, we assume that the measurable scheduling signal $K(t)$ is of the form of Fig. 6(a).

A hysteresis switching scheme over the robust controller set $\mathcal{C}$ based on the work of Yan and Özbay (2007) is presented in Table 1, where we have chosen $|d_{\eta,\eta+1}|$ to be 0.3. In our case, the dwell time is $\tau_D = 0.096\ h$. This mean we can allow $\omega_0 = \sup_{t\geq 0} |\dot{K}(t)|$ to be in the range of $\omega_0 \in (0, 3.125)$. Also note that $max\{\dot{K}(t)\} = \frac{4\pi}{3000} < \frac{|d_{\eta,\eta+1}|}{\tau_D} = 3.125$, which concludes that the switching LPV system is stable with $\mathcal{C}$ according to (18).

Table 1. Hysterisis Switching Scheme

Switching Logic	@ Value of K
Switch: $C_1 \rightarrow C_2$	@ $K = -4.35$
Switch: $C_1 \leftarrow C_2$	@ $K = -4.65$
Switch: $C_2 \rightarrow C_3$	@ $K = -2.15$
Switch: $C_2 \leftarrow C_3$	@ $K = -2.45$
Switch: $C_3 \rightarrow C_4$	@ $K = -1.15$
Switch: $C_3 \leftarrow C_4$	@ $K = -1.45$
Switch: $C_4 \rightarrow C_5$	@ $K = -0.65$
Switch: $C_4 \leftarrow C_5$	@ $K = -0.95$
Switch: $C_5 \rightarrow C_6$	@ $K = -0.30$
Switch: $C_5 \leftarrow C_6$	@ $K = -0.60$

The simulations results are shown in Fig. 6(b) and (c). Fig. 6(b) verifies the performance of our switching robust control scheme under nominal conditions i.e. $\alpha_0 = 0.5$ and $T_0 = 50$ with an action delay in switching of $\tau_{switch} = 60$ *sec*. We observe that our control scheme fulfills all the required performance specifications i.e. settling time, undershoot and steady state performance mentioned in section 2.2 with a reasonable infusion of vasodilator drug in the acceptable range of $0 \leq I(t) \leq 80\ ml\ h^{-1}$. Fig. 6(c) shows the performance of our control algorithm under worst case scenario i.e. $\alpha_0 = 0.65$ and $T_0 = 60$ with a large switching action delay of $\tau_{switch} = 45$ *sec*. Simulations confirm the performance of our designed scheme even under extreme conditions with a relatively large switching action delay, satisfying all design requirements with drug infusion in the acceptable range of $0 \leq I(t) \leq 80\ ml\ h^{-1}$.

6. CONCLUSION

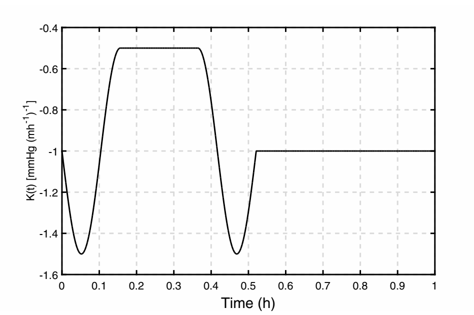
This paper is an application of the work of Yan and Özbay (2007) to the problem of regulating postsurgical hypertension using infusion of vasodilator drug. In the extended version of this paper, the proposed method will be compared with the most recent LPV controller results in the literature, Luspay and Grigoriadis (2015), on the problem under consideration. We also plan to use the actual clinical data for on-line measurement of the scheduling parameter using a MMEKF algorithm. Finally, the sufficient condition for stability provided in terms of bound on the parameter variation by Corollary 3.1 of Yan and Özbay (2007) is conservative. So as an extension to this work, we also plan to apply less conservative results based on average dwell time to allow for faster parameter variation and faster switching.

REFERENCES

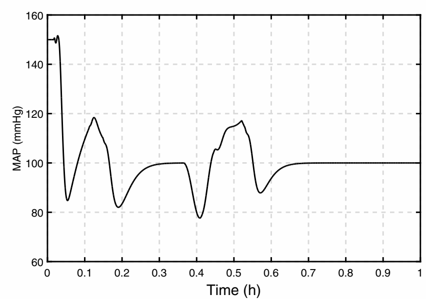
Ahmed, S. and Özbay, H. (2015). Switching robust controllers for automatic regulation of postoperative hypertension using vasodilator drug infusion rate. *1st IFAC Workshop on Linear Parameter Varying Systems*, 48(26), 224–229.

De Asla, R., Benis, A., Jurado, R., and Litwak, R. (1985). Management of postcardiotomy hypertension by microcomputer-controlled administration of sodium nitroprusside. *The Journal of Thoracic and Cardiovascular Surgery*, 89(1), 115–120.

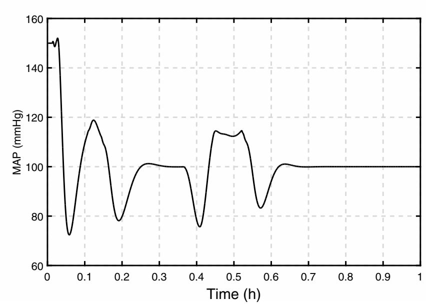
Isaka, S. and Sebald, A. (1993). Control strategies for arterial blood pressure regulation. *IEEE Transactions on Biomedical Engineering*, 40(4), 353–363.



(a) Measurable Scheduling Signal, $K(t)$



(b) MAP under nominal conditions with a switching action delay



(c) MAP under extreme conditions with a switching action delay

Fig. 6. Scheduling Signal $K(t)$ $[mmHg\left(m\ h^{-1}\right)^{-1}]$ and curves of MAP $(mmHg)$ under various conditions

Koch-Weser, J. (1974). Vasodilator drugs in the treatment of hypertension. *Archives of Internal Medicine*, 133(6), 1017–1027.

Luspay, T. and Grigoriadis, K.M. (2014). Design and validation of an extended kalman filter for estimating hemodynamic variables. *American Control Conference (ACC)*, 4145–4150.

Luspay, T. and Grigoriadis, K. (2015). Robust linear parameter varying control of blood pressure using vasoactive drugs. *International Journal of Control*, 88(10), 2013–2029.

Malagutti, N. (2014). Particle filter-based robust adaptive control for closed-loop administration of sodium nitroprusside. *Journal of Computational Surgery*, 1(1), 1–19.

Malagutti, N., Dehghani, A., and Kennedy, R.A. (2013). Robust control design for automatic regulation of blood pressure. *IET Control Theory & Applications*, 7(3), 387–396.

Martin, J.F., Schneider, A.M., and Smith, N.T. (1987). Multiple-model adaptive control of blood pressure using sodium nitroprusside. *IEEE Transactions on Biomedical Engineering*, (8), 603–611.

Meijers, R.H., Schmartz, D., Cantraine, F.R., Barvais, L., dHollander, A.A., and Blom, J.A. (1997). Clinical evaluation of an automatic blood pressure controller during cardiac surgery. *Journal of Clinical Monitoring*, 13(4), 261–268.

Mitchell, R.R. (1982). The need for closed-loop therapy. *Critical Care Medicine*, 10(12), 831–834.

Ozbay, H. (1999). *Introduction to Feedback Control Theory.* CRC Press.

Pardini, B.J., Lund, D.D., Wurster, R.D., and Anderson, R.H. (1988). An electronic, negative feedback device to control arterial pressure. *American Journal of Physiology-Heart and Circulatory Physiology*, 254(1), H187–H191.

Sheppard, L., Shotts, J., Roberson, N., Wallace, F., and Kouchoukos, N. (1979). Computer controlled infusion of vasoactive drugs in post cardiac surgical patients. *IEEE/1979 Frontiers of Engineering in Health Care (IEEE CH1440-7)*, 280–284.

Slate, J., Sheppard, L., Rideout, V., and Blackstone, E. (1979). Model for design of a blood-pressure controller for hypertensive patients. *IEEE Transactions on Biomedical Engineering*, 26(9), 541–541.

Smolen, V., Barile, R., and Carr, D. (1979). Design and operation of a system for automatic feedback-controlled administration of drugs. *Medical Device and Diagnostic Industry*, 1, 51–69.

Wood, M., Hyman, S., and Wood, A.J. (1987). A clinical study of sensitivity to sodium nitroprusside during controlled hypotensive anesthesia in young and elderly patients. *Anesthesia & Analgesia*, 66(2), 132–136.

Yan, P. and Özbay, H. (2007). On switching $\mathcal{H}^\infty$ controllers for a class of linear parameter varying systems. *Systems & Control Letters*, 56(7), 504–511.

Zhou, K., Doyle, J.C., Glover, K., (1996). *Robust and Optimal Control*, Prentice Hall, New Jersey.